JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

COMPLEX NUMBER

(PRACTICE SHEET)

LEVEL- 1

Question based on Imaginary Numbers

Q.1 $i^{57} + 1/i^{125}$ is equal to-

(A) 0 (B) –2i

(C) 2i (D) 2

Q.2 $\{1 + (-i)^{4n+3}\}\ (1 - i)\ (n \in N)$ equals-

(A) 2 (B) – 1

(C) – 2 (D) i

Q.3 $\left(\frac{-1-i}{\sqrt{2}}\right)^{100}$ equals-

(A) 1 (B) – i

(C) i (D) – 1

Q.4 The value of $(-i)^{-117}$ is-

(A) – 1 (B) i

(C) 1 (D) – i

Q.5 $(i^{10} + 1)(i^{9} + 1)\ (i^{8} + 1)..........(i + 1)$ equals-

(A) –1 (B) 1

(C) i (D) 0

Q.6 i^{243} equals -

(A) – 1 (B) 1

(C) i (D) – i

Q.7 $\frac{1+i^2+i^3+i^4+i^5}{1+i}$ equals-

(A) 1 – i (B) (1 + i)/2

(C) (1 – i)/2 (D) 1 + i

Q.8 If $k \in N$, then $\frac{i^{4k+1} - i^{4k-1}}{2}$ is equal to-

(A) –1 (B) i

(C) 1 (D) – i

Q.9 The value of $(1 + i)^{2n} + (1 - i)^{2n}$ $(n \in N)$ is zero, if-

(A) n is odd (B) n is multiple of 4

(C) n is even (D) $\frac{n}{2}$ is odd

Q.10 The value of the expression $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ is-

(A) 0 (B) 1 (C) – 1 (D) –2

Question based on Complex Number

Q.11 The real and imaginary parts of $\frac{5+3i}{i-2}$ are-

(A) –5 / 2, 3 (B) –1, – 3/ 5

(C) –7 / 5, –11/5 (D) 7 / 5, –11/5

Q.12 The value of $\frac{1}{1-i} - \frac{1}{1+i}$ is-

(A) purely rational (B) purely imaginary

(C) purely real (D) None of these

Q.13 The conjugate of $\frac{(2+i)^2}{3+4i}$ is-

(A) 1 (B) purely imaginary

(C) – 1 (D) None of these

Q.14 $(x, y)^2$ is equal to-

(A) $(x^2 - y^2, 0)$ (B) $(x^2 - y^2, 2xy)$

(C) (x^2, y^2) (D) (2x, 2y)

Q.15 The conjugate of $\frac{3+2i}{5-3i}$ is equal to-

(A) $-\frac{1}{34}(9 + 19i)$ (B) $\frac{1}{34}(9 - 19i)$

(C) $\frac{1}{34}(19i - 9)$ (D) $\frac{1}{34}(9 + 19i)$

Q.16 If $z^2 = (\bar{z})^2$, then which statement is true -

(A) z is imaginary (B) z is real

(C) $z = -\bar{z}$ (D) z is real or imaginary

Q.17 If $z = \cos\theta + i\sin\theta$, then $\frac{1+z}{1-z}$ is equal to

(A) $i\tan\theta$ (B) $i\cot\theta/2$

(C) $i\cot\theta$ (D) $i\tan\theta/2$

Q.18 If $I\left(\frac{2z+1}{iz+1}\right) = -2$, then the locus of z is -

(A) a parabola (B) a straight line
(C) a circle (D) a coordinate axis

Q.19 Which of the following is a complex number

(A) $\left(\tan\pi, \tan\frac{\pi}{2}\right)$ (B) $(\sqrt{e}, i^8)$
(C) $(0, \sqrt{-1})$ (D) None of these

Q.20 Which one is a complex number?

(A) (i^4, i^5) (B) (i^8, i^{12})
(C) $(\sqrt{-4}, 4)$ (D) $\{\log 2, \log(-1)\}$

Q.21 Which of the following is the correct statement?

(A) $1 - i < 1 + i$ (B) $2i > i$
(C) $2i + 1 > -2i + 1$ (D) None of these

Q.22 $a + ib > c + id$ is meaningful if-

(A) a = 0, d = 0 (B) a = 0, c = 0
(C) b = 0, c = 0 (D) d = 0, b = 0

Q.23 The number $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$ is-

(A) zero (B) purely real
(C) purely imaginary (D) complex

Q.24 If $\sqrt{x}\,(i + \sqrt{y}) - 15 = i\,(8 - \sqrt{y})$. Then x & y equals to-

(A) 25, 5 (B) 25, 9
(C) 9, 5 (D) 5, 16

Q.25 If $(x + iy)(2 - 3i) = 4 + i$, then-

(A) $x = -\frac{5}{13}$, $y = \frac{14}{13}$ (B) $x = \frac{5}{13}$, $y = -\frac{14}{13}$
(C) $x = \frac{14}{13}$, $y = \frac{5}{13}$ (D) $x = \frac{5}{13}$, $y = \frac{14}{13}$

Q.26 The value of x and y which satisfies the equation $\frac{(1+i)^2}{(1-i)^2} + \frac{1}{x+iy} = 1 + i$ is-

(A) $x = \frac{2}{5}$, $y = -\frac{1}{5}$ (B) $x = -\frac{2}{5}$, $y = -\frac{1}{5}$
(C) $x = -\frac{2}{5}$, $y = \frac{1}{5}$ (D) $x = \frac{2}{5}$, $y = \frac{1}{5}$

Q.27 If $z = -3 + 2i$, then $1/z$ is equal to-

(A) $-\frac{1}{13}(3 + 2i)$ (B) $\frac{1}{13}(3 + 2i)$
(C) $\frac{1}{\sqrt{13}}(3 + 2i)$ (D) $-\frac{1}{\sqrt{13}} - (3 + 2i)$

Q.28 If $2\sin\theta - 2i\cos\theta = 1 + i\sqrt{3}$, then value of θ is-

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q.29 If $z_1, z_2 \in C$, then which statement is true?

(A) $R(z_1 - z_2) = R(z_1) - R(z_2)$
(B) $R(z_1 / z_2) = R(z_1) / R(z_2)$
(C) $R(z_1 z_2) = R(z_1)\,R(z_2)$
(D) None of these

Q.30 If $z_1, z_2 \in C$, then wrong statement is-

(A) $\overline{z_1 + z_2} = \bar{z}_2 + \bar{z}_1$
(B) $|z_1\,\bar{z}_2| = |z_2|\,|z_1|$
(C) $\overline{z_1 z_2} = \bar{z}_2\,\bar{z}_1$
(D) $|z_1 + \bar{z}_2| = |z_1 - \bar{z}_2|$

Q.31 If $z = x + iy$, then $\frac{z - \bar{z}}{z + \bar{z}}$ is equal to-

(A) i(y/x) (B) y/x
(C) i(x/y) (D) x/y

Q.32 For any complex number z which statement is true-

(A) $z - \bar{z}$ is purely real number
(B) $z + \bar{z}$ is purely imaginary number
(C) $z\bar{z}$ is purely imaginary number
(D) $z\bar{z}$ is non-negative real number

Q.33 If z and $\bar{z}$ are equal then locus of the point z in the complex plane is

(A) real axis (B) circle
(C) imaginary axis (D) None of these

Q.34 If $c^2 + s^2 = 1$, then $\frac{1+c+is}{1+c-is} =$

(A) c + is (B) s + ic
(C) c – is (D) s – ic

Q.35 For any complex number z, $\bar{z} = (1/z)$, if -
(A) z is purely imaginary
(B) $|z| = 1$
(C) z is purely real
(D) $z = 1$

Q.36 If $z = 1 + i$, then multiplicative inverse of z^2 is-
(A) 2i (B) $-i/2$
(C) $i/2$ (D) $1 - i$

Question based on **Modulus of a Complex Number**

Q.37 The modulus of complex number $z = -2i(1-i)^2(1+i\sqrt{3})^3$ is-
(A) 32 (B) 0
(C) – 32 (D) 1

Q.38 The modulus of sum of complex numbers $-4 + 3i$ and $-8 + 6i$ is-
(A) equal to sum of moduli
(B) greater than or equal to sum of moduli
(C) less than or equal to sum of moduli
(D) none of these

Q.39 If $z_1 = 2 + i$, $z_2 = 3 - 2i$, then value of $\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2$ is -
(A) 2 (B) 1
(C) 0 (D) None of these

Q.40 Modulus of $\frac{\cos\theta - i\sin\theta}{\sin\theta - i\cos\theta}$ is-
(A) 0 (B) 2θ
(C) $\pi - 2\theta$ (D) None of these

Q.41 If $z = x + iy$ & $|z - 3| = R(z)$, then locus of z is-
(A) $y^2 = -3(2x + 3)$ (B) $y^2 = 3(2x + 3)$
(C) $y^2 = -3(2x - 3)$ (D) $y^2 = 3(2x - 3)$

Q.42 If z_1 and z_2 are any two complex numbers, then $\frac{|z_2 + z_1|}{||z_2| - |z_1||}$ is-
(A) ≥ 1 (B) ≥ 1
(C) ≥ -1 (D) None of these

Q.43 If $|z| + 2 = I(z)$, then $z = (x, y)$ lies on-
(A) $y^2 = -4(x-1)$ (B) $y^2 = 4(x-1)$
(C) $x^2 = -4(y-1)$ (D) No locus

Q.44 The complex number z which satisfy the condition $|z| + z = 0$ always lie on-
(A) y-axis (B) x-axis
(C) x-axis and $x < 0$ (D) $x = y$

Q.45 If $(-7 - 24i)^{1/2} = x - iy$, then $x^2 + y^2$ is equal to-
(A) $\sqrt{25}$ (B) 25
(C) 15 (D) None of these

Q.46 If z_1 and z_2 be two complex numbers, then which statement is true-
(A) $|z_1 + z_2| \leq |z_1| + |z_2|$
(B) $|z_1 - z_2| = |z_1| + |z_2|$
(C) $|z_1 + z_2| \geq |z_1 - z_2|$
(D) $|z_1 + z_2| \geq |z_1| + |z_2|$

Q.47 If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$, then $a^2 + b^2 =$
(A) 2 (B) 1 (C) 3 (D) 4

Question based on **Amplitude of a Complex Number**

Q.48 If amp $(z_i) = \theta_i$, i = 1, 2, 3; then amp $\left(\frac{z_1}{z_2\bar{z}_3}\right)$ is equal to-
(A) $\frac{\theta_1}{\theta_2\theta_3}$ (B) $\frac{\theta_1\theta_2}{\theta_3}$
(C) $\theta_1 - \theta_2 - \theta_3$ (D) $\theta_1 - \theta_2 + \theta_3$

Q.49 The amplitude of $-1 - i\sqrt{3}$ is-
(A) $-\pi/3$ (B) $\pi/3$
(C) $2\pi/3$ (D) $-2\pi/3$

Q.50 The amplitude of $\sin\frac{6\pi}{5} + i\left(1 + \cos\frac{6\pi}{5}\right)$ is-
(A) $3\pi/5$ (B) $9\pi/10$
(C) $3\pi/10$ (D) None of these

Q.51 The amplitude of $3 - \sqrt{8}$ is-
(A) 0 (B) $\pi/2$
(C) π (D) $-\pi/2$

Q.52 The amplitude of 1/i is equal to-

(A) π (B) $\pi/2$

(C) $-\pi/2$ (D) 0

Q.53 If amp (z) = θ, then amp (1/z) is equal to-

(A) θ (B) $-\theta$

(C) $\pi - \theta$ (D) $\pi + \theta$

Q.54 The amplitude of $1 - \cos\theta - i\sin\theta$ is-

(A) $\pi + (\theta/2)$ (B) $(\pi - \theta)/2$

(C) $(\theta - \pi)/2$ (D) $\theta/2$

Q.55 The amplitude of complex number

$z = \frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is-

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{2}$

Q.56 If $z = \frac{(1+i)\sqrt{3}-(1-i)}{2\sqrt{2}}$, then -

(A) $|z| = 1$, amp (z) = $-\pi/4$

(B) $|z| = 1$, amp (z) = $\pi/4$

(C) $|z| = 1$, amp (z) = $5\pi/12$

(D) $|z| = 1$, amp (z) = $\pi/12$

Q.57 The amplitude of $\frac{(1+i)(2+i)}{3-i}$ is-

(A) $-\pi/3$ (B) $\pi/2$

(C) $\pi/3$ (D) $-\pi/2$

Q.58 If z_1, z_2 are two complex numbers such that $|z_1 + z_2| = |z_1 - z_2|$ then $\text{amp}(z_1) - \text{amp}(z_2)$ is equal to-

(A) $\pi/3$ (B) $\pi/2$

(C) $\pi/4$ (D) 0

Q.59 If amp(z) = α, then amp (iz) is equal to-

(A) $\pi - \alpha$ (B) $(\pi/2) + \alpha$

(C) $(\pi/2) - \alpha$ (D) $-\alpha$

Q.60 The amplitude of complex number $(1 + i\sqrt{3})(1 + i)(\cos\theta + i\sin\theta)$ is-

(A) $\frac{\pi}{12} - \theta$ (B) $\frac{7\pi}{12} + \theta$

(C) $\frac{7\pi}{12} - \theta$ (D) $\frac{\pi}{12} + \theta$

Q.61 If z_1 and z_2 are two conjugate complex numbers and amp $(z_1) = \theta$, then amp (z_1) + amp (z_2) and amp (z_1/z_2) are equal to -

(A) $2\theta, -2\theta$ (B) $0, 2\theta$

(C) $2\theta, 0$ (D) None of these

Q.62 The amplitude of $\left|\frac{x+iy}{x-iy}\right|$ is -

(A) $\tan^{-1}(y/x)$ (B) $2\tan^{-1}(y/x)$

(C) 0 (D) $\pi/2$

Q.63 amp $(\cot\alpha - i)$ equals-

(A) $(\pi/2) + \alpha$ (B) $-\alpha$

(C) α (D) $\alpha - (\pi/2)$

Q.64 The arg of $\frac{1}{4}(1 - i\sqrt{3})^2$ is-

(A) $2\pi/3$ (B) $-2\pi/3$ (C) 2π (D) π

Q.65 If $\sqrt{3} + i = (a + ib)(c + id)$, then

$\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right) =$

(A) $n\pi - \frac{\pi}{3}$ (B) $n\pi + \frac{\pi}{6}$

(C) $\frac{\pi}{3} + 2n\pi$ (D) $2n\pi - \frac{\pi}{3}$

Q.66 If amplitude of $\frac{2+i}{i-1}$ is θ, then -

(A) $0 < \theta < \pi/2$ (B) $-\pi/2 < \theta < 0$

(C) $\pi/2 < \theta < \pi$ (D) $-\pi < \theta < -\pi/2$

Question based on Polar form of Complex Number

Q.67 The polar form of $-5(\cos 40^\circ - i\sin 40^\circ)$ is-

(A) $5(\cos 140^\circ + i\sin 140^\circ)$

(B) $5(\cos 140^\circ - i\sin 140^\circ)$

(C) $5(\cos 40^\circ - i\sin 40^\circ)$

(D) $5(\cos 40^\circ + i\sin 40^\circ)$

Q.68 The polar form of $\frac{1+7i}{(2-i)^2}$ is -

(A) $\sqrt{2}\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$

(B) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(C) $\sqrt{2}\left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)$

(D) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Q.69 r (cos θ + i sin θ) form of $\frac{1-i}{1+i}$ is -

(A) $\sin\frac{\pi}{2} + i\cos\frac{\pi}{2}$ (B) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

(C) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (D) None of these

Q.70 – 3 – 4i equals-

(A) $5e^{i\{\pi-\tan^{-1}(3/4)\}}$ (B) $5e^{-i\{\pi-\tan^{-1}(4/3)\}}$

(C) $5e^{i\{\pi-\tan^{-1}(4/3)\}}$ (D) $5e^{i\{\pi-\tan^{-1}(3/4)\}}$

Q.71 If modulus and amplitude of a complex number are 2 and 2π/3 respectively, then the number is-

(A) $1 - i\sqrt{3}$ (B) $1 + i\sqrt{3}$

(C) $-1 + i\sqrt{3}$ (D) $-1 - i\sqrt{3}$

Question based on **Square root of Complex Number**

Q.72 The square root of –5 –12i is-

(A) ± (3 – 2i) (B) ± (2 – 3i)

(C) ± (3 + 2i) (D) ± (2 – i)

Q.73 The square root of 8 – 6i is-

(A) ± (1 + 3i) (B) ± (3 – i)

(C) ± (1 – 3i) (D) ± (3 + i)

Q.74 The square root of i is-

(A) $\pm\frac{1}{\sqrt{2}}(1+i)$ (B) $\pm\frac{1}{\sqrt{2}}(1-i)$

(C) $\pm\sqrt{2}(1-i)$ (D) $\pm\sqrt{2}(1+i)$

Q.75 The square root of –7 + 24i is-

(A) ± (3 + 4i) (B) ± (–3 + 4i)

(C) ± (–4 + 3i) (D) ± (4 + 3i)

Question based on **Cube roots of unity**

Q.76 If ω is cube root of unity, then the value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ is-

(A) 1 (B) 0

(C) – 1 (D) 2

Q.77 The value of $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n$ is-

(A) $2^n \sin n\pi/6$ (B) $2^n \cos n\pi/6$

(C) $2^{n+1} \cos n\pi/6$ (D) $2^{n+1} \sin n\pi/6$

Q.78 If ω is cube root of unity and if n = 3k + 2 then the value of $\omega^n + \omega^{2n}$ is-

(A) 0 (B) –1 (C) 2 (D) 1

Q.79 If ω is cube root of unity then the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ ...... 2n is-

(A) 0 (B) n (C) –1 (D) 1

Q.80 $\left[\frac{-1+i\sqrt{3}}{2}\right]^6 + \left[\frac{-1-i\sqrt{3}}{2}\right]^6 + \left[\frac{-1+i\sqrt{3}}{2}\right]^5 + \left[\frac{-1-i\sqrt{3}}{2}\right]^5 =$

(A) 1 (B) –1

(C) 2 (D) None of these

Q.81 If ω is cube root of unity, then the value of $(1+\omega) - (1-\omega^2) - 3(1+\omega^2)^3$ is-

(A) 0 (B) 1 (C) –1 (D) 2

Q.82 If $x^3 - 1 = 0$ has the non-real complex roots α, β then the value of $(1+2\alpha+\beta)^3 - (3+3\alpha+5\beta)^3$ is:

(A) – 4 (B) 6 (C) –7 (D) 0

Q.83 If ω is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}....\right)}$ equals-

(A) –1 (B) 0 (C) 9 (D) i

Q.84 If ω is a non real cube root of unity and n is a positive integer which is not a multiple of 3; then $1 + \omega^n + \omega^{2n}$ is equal to-

(A) 3ω (B) 0

(C) 3 (D) None of these

Q.85 The sum of squares of cube roots of unity is-

(A) 0 (B) – 1

(C) 1 (D) 3

Q.86 If x = a + b, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$, then xyz equals-

(A) $(a+b)^3$ (B) $a^3 - b^3$

(C) $(a+b)^3 + 3ab(a+b)$ (D) $a^3 + b^3$

Q.87 The cube roots of unity-
(A) form an equilateral Δ
(B) are all complex numbers
(C) lie on the circle $|Z| = 1$
(D) All of these

Question based on Geometry of Complex Number

Q.88 If $z = (k + 3) + i\sqrt{5-k^2}$, then locus of z is a-
(A) circle (B) parabola
(C) straight line (D) None of these

Q.89 If $\bar{z} = 2 - z$, then locus of z is a
(A) line passing through origin
(B) line parallel to y-axis
(C) line parallel to x-axis
(D) circle

Q.90 The value of z for which $|z + i| = |z - i|$ is-
(A) any real number
(B) any natural number
(C) any complex number
(D) None of these

Q.91 If $|z| = 2$, then locus of $-1 + 5z$ is a circle whose centre is-
(A) $(-1, 0)$ (B) $(1, 0)$
(C) $(0, -1)$ (D) $(0, 0)$

Q.92 If centre of any circle is at point z_1 and its radius is a, then its equation is-
(A) $|z + z_1| = a$ (B) $|z| = a$
(C) $|z - z_1| < a$ (D) $|z - z_1| = a$

Q.93 If 0, 3 + 4i, 7 + 7i, 4 + 3i are vertices of a quadrilateral, then its, is-
(A) square (B) rectangle
(C) parallelogram (D) rhombus

Q.94 If complex numbers z_1, z_2, z_3 represent the vertices A, B, C of a parallelogram ABCD respectively, then the vertex D is -
(A) $\frac{1}{2}(z_1 + z_2 - z_3)$ (B) $\frac{1}{2}(z_1 + z_2 + z_3)$
(C) $z_1 + z_3 - z_2$ (D) $2(z_1 + z_2 - z_3)$

Q.95 If complex numbers 2i, 5 + i and 4 represent points A, B and C respectively, then centroid of ΔABC is-
(A) 2 + i (B) 1 + 3i
(C) 3 + i (D) 3 – i

Q.96 If complex numbers 1, –1 and $\sqrt{3}\,i$ are represented by points A, B and C respectively on a complex plane, then they are-
(A) vertices of an isosceles triangle
(B) vertices of right-angled triangle
(C) collinear
(D) vertices of an equilateral triangle

Q.97 If 1 + 2i, – 2 + 3i, – 3 – 4i are vertices of a triangle, then its area is-
(A) 11 (B) 22
(C) 16 (D) 30

Q.98 The length of a straight line segment joining complex numbers 2 and –3i is-
(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) $\sqrt{13}$ (D) 13

Q.99 If $z = x + iy$, then $I(z) > 0$ represents a region-
(A) above real axis
(B) below real axis
(C) right of imaginary axis
(D) None of these

Q.100 If $|z| = 3$, then point represented by $2 - z$ lie on the circle-
(A) centre (2, 0), radius = 3
(B) centre (0, 2), radius = 3
(C) centre (2, 0), radius = 1
(D) None of these

Q.101 $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ is the equation of a circle, if -
(A) $|a|^2 < b$ (B) $|a|^2 \geq b$
(C) $|a|^2 \leq b$ (D) None of these

Q.102 If z is a complex number, then radius of the circle $z\bar{z} - 2(1 + i)z - 2(1 - i)\bar{z} - 1 = 0$ is-
(A) 2 (B) 1
(C) 3 (D) 4

LEVEL- 2

Q.1 If $|z_1| = |z_2| .... = |z_n| = 1$, then $\left|\frac{z_1 + z_2 + ... + z_n}{z_1^{-1} + z_2^{-1} + ... + z_n^{-1}}\right|$ equals-

(A) 1/n (B) n
(C) 1 (D) $|z_1 + z_2 + ... + z_n|$

Q.2 If $\alpha = \cos\theta + i\sin\theta$, then $\frac{1+\alpha}{1-\alpha}$ equals -

(A) $\cot\theta$ (B) $i\tan\frac{\theta}{2}$
(C) $i\cot\frac{\theta}{2}$ (D) $\cot\frac{\theta}{2}$

Q.3 If $(1 + i)(1 + 2i).......(1 + ix) = a + ib$, then $2.5........(1 + x^2)$ equals -

(A) a + b (B) a – b
(C) $a^2 + b^2$ (D) $a^2 - b^2$

Q.4 If $z + \sqrt{2}\,|z + 1| + i = 0$, then z equals-

(A) 2 + i (B) – 2 + i
(C) $-\frac{1}{2} + i$ (D) – 2 – i

Q.5 If $(2 + i)r^{-1} = \{4i + (1 + i)^2\}(\cos\theta + i\sin\theta)$, then value of | r | is -

(A) $\sqrt{(5/6)}$ (B) $\sqrt{5}/6$
(C) 5/6 (D) None of these

Q.6 Modulus of $1 + i\tan\alpha$ ($\frac{\pi}{2} < \alpha < \pi$) is -

(A) $\text{cosec}\,\alpha$ (B) $\sec\alpha$
(C) $-\frac{1}{\cos\alpha}$ (D) None of these

Q.7 If $-3 + ix^2y$ is the conjugate of $x^2 + y + 4i$, then real values of x and y are-

(A) x = ± 1, y = 1 (B) x = – 1, y = –4
(C) x = 1, y = –4 (D) x = ±1, y = –4

Q.8 If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, then θ is equal to-

(A) $2n\pi \pm \pi/3$ (B) $n\pi \pm \pi/3$
(C) $n\pi \pm \pi/6$ (D) $2n\pi \pm \pi/6$

Q.9 If $\sqrt{a+ib} = (\alpha + i\beta)$ then $\sqrt{-a-ib} =$

(A) $-(\alpha + i\beta)$ (B) $i(\alpha - i\beta)$
(C) $\pm(\beta - i\alpha)$ (D) $\pm(\alpha + i\beta)$

Q.10 For any two non zero complex numbers z_1 and z_2 if $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$, then amp (z_1) – amp(z_2) is -

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Q.11 $(x + iy)^{1/3} = a + ib$, then $\frac{x}{a} + \frac{y}{b}$ is equal to-

(A) 0 (B) – 1
(C) 1 (D) None of these

Q.12 If z_1, z_2 are complex numbers such that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then z_1 / z_2 is-

(A) zero (B) purely imaginary
(C) purely real (D) None of these

Q.13 If $z = \sqrt{2i}$, then z is equal to-

(A) $\pm\frac{1}{\sqrt{2}}(1 + i)$ (B) $\pm\frac{1}{\sqrt{2}}(1 - i)$
(C) ± (1 – i) (D) ± (1 + i)

Q.14 Vector z = 3 – 4i is rotated at 180º angle in anti clockwise direction and its length is increased to two and half times. In new position, z is -

(A) (15/2) + 10i (B) –(15/2) + 10i
(C) – 15 + 10i (D) None of these

Q.15 If the first term and common ratio of a G.P. is $\frac{1}{2}(\sqrt{3} + i)$, then the modulus of its nth term will be-

(A) 1 (B) 2^{2n} (C) 2^n (D) 2^{3n}

Q.16 The least positive value of n for which $\left[\frac{i(i+\sqrt{3})}{1-i^2}\right]^n$ is a positive integer is -

(A) 2 (B) 1 (C) 3 (D) 4

Q.17 If $\frac{z^2}{(z-1)}$ is always real, then locus of z is -

(A) real axis (B) circle
(C) imaginary axis (D) real axis or a circle

Q.18 If z (≠ 2) be a complex numbers such that $\log_{1/2}|z - 2| > \log_{1/2}|z|$, then z satisfies -

(A) Re(z) < 1 (B) Re (z) > 1
(C) Im (z) = 1 (D) Im (z) < 1

Q.19 If $\left|\frac{z-a}{z+\bar{a}}\right| = 1$, Re(a) ≠ 0, then locus of z is-

(A) x = |a| (B) imaginary axis
(C) real axis (D) None of these

Q.20 If z = x + iy, then the equation $\left|\frac{2z-i}{z+1}\right| = k$ will be a straight line, where -

(A) k = 1 (B) k = 1/2
(C) k = 2 (D) k = 3

Q.21 The slope of the line $|z - 1| = |z + i|$ is-
(A) 2 (B) 1/2 (C) – 1 (D) 0

Q.22 If $z_1, z_2 \in C$ such that $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$, then z_1/z_2 is-
(A) negative real number
(B) positive real number
(C) zero or purely imaginary
(D) None of these

Q.23 If $z = x + iy$ and $|z - 1 + 2i| = |z + 1 - 2i|$, then the locus of z is -
(A) $x + y = 0$ (B) $x = y$
(C) $x = 2y$ (D) $x + 2y = 0$

Q.24 If $z = x + iy$ and amp $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$, then locus of z is -
(A) a parabola (B) a straight line
(C) a circle (D) x–axis

Q.25 If $|z - i| = 1$ and amp(z) $= \pi/2$ $(z \neq 0)$, then z is-
(A) – 2i (B) (2, 0) (C) 2i (D) 1 + i

Q.26 The locus of a point z in complex plane satisfying the condition $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{2}$ is -
(A) a circle with centre (0, 0) and radius 2
(B) a straight line
(C) a circle with centre (0, 0) and radius 3
(D) None of these

Q.27 If z is a complex number, then amp $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ will be-
(A) $|z| = 1$, R(z) > 0 (B) $|z| = 1$
(C) $|z| = 1$, I(z) < 0 (D) $|z| = 1$, I(z) > 0

Q.28 If $z = x + iy$, then $1 \leq |z| \leq 3$ represents-
(A) a circular region
(B) region between two lines parallel to imaginary axis
(C) region between two lines parallel to real axis
(D) region between two concentric circles

Q.29 The triangle formed by z, iz and i^2z is-
(A) right-angled
(B) equilateral
(C) isosceles
(D) right-angled isosceles

Q.30 The centre of a square is at the origin and one of the vertex is 1 – i. The extremities of diagonal not passing through this vertex are-
(A) 1 + i, –1 – i (B) –1 + i, –1 –i
(C) 1 + i, –1 + i (D) None of these

Q.31 If z_1, z_2 are two complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then origin and z_1, z_2 are vertices of a triangle which is -
(A) equilateral (B) right angled
(C) isosceles (D) None of these

Q.32 The number of solutions of the system of equations $\text{Re}(z^2) = 0$, $|z| = 2$ is -
(A) 4 (B) 2 (C) 3 (D) 1

Q.33 If z_1, z_2, z_3, z_4 are any four points in a complex plane and z is a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then z_1, z_2, z_3, and z_4, are-
(A) vertices of a rhombus
(B) vertices of a rectangle
(C) concyclic
(D) collinear

Q.34 Let z be a complex number satisfying $|z - 5i| \leq 1$ such that amp(z) is minimum, then z is equal to-
(A) $\frac{2\sqrt{6}}{5} + \frac{24}{5}i$ (B) $\frac{2\sqrt{6}}{5} - \frac{24}{5}i$
(C) $\frac{24}{5} + \frac{2\sqrt{6}}{5}i$ (D) None of these

Q.35 The system of equations $|z + 2 - 2i| = 4$ and $|z| = 1$ has -
(A) two solutions (B) one solution
(C) infinite solutions (D) no solution

Q.36 In the region $|z + 1 - i| \leq 1$ which of the following complex number has least positive argument-
(A) i (B) 1 + i (C) – i (D) – 1 + i

Q.37 If $\left|z - \frac{4}{z}\right| = 4$, then the greatest value of $|z|$ is-
(A) $2\sqrt{2}$ (B) $2(\sqrt{2} + 1)$
(C) $2(\sqrt{2} - 1)$ (D) None of these

LEVEL- 3

Q.1 If the area of the triangle on the complex plane formed by complex numbers z, ωz and z + ωz is $4\sqrt{3}$ square units, then |z| is-

(A) 4 (B) 2
(C) 6 (D) 3

Q.2 If $\frac{5z_2}{7z_1}$ is purely imaginary, then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is equal to-

(A) 5/7 (B) 7/9
(C) 25/49 (D) none of these

Q.3 If the complex numbers $z_1 = a + i$, $z_2 = 1 + ib$, $z_3 = 0$ form an equilateral triangle (a, b are real numbers between 0 and 1), then-

(A) $a = \sqrt{3} - 1$, $b = \frac{\sqrt{3}}{2}$
(B) $a = 2 - \sqrt{3}$, $b = 2 - \sqrt{3}$
(C) a = 1/2, b = 3/4
(D) None of these

Q.4 The minimum value of |2 z – 1| + |3z –2| is-

(A) 0 (B) 1/2
(C) 1/3 (D) 2/3

Q.5 The centre of a regular hexagon is i. One vertex is (2 + i), z is an adjacent vertex. Then z =

(A) $1 + i(1 \pm \sqrt{3})$ (B) $i + 2 \pm \sqrt{3}$
(C) $2 + i(1 \pm \sqrt{3})$ (D) None of these

Q.6 If $z_1 = 1 + 2i$, $z_2 = 2 + 3i$, $z_3 = 3 + 4i$, then z_1, z_2 and z_3 represent the vertices of -

(A) equilateral triangle
(B) right angled triangle
(C) isosceles
(D) None of these

Q.7 The value of the expression

$$\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+\ldots\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right),$$

where ω is an imaginary cube root of unity is-

(A) $\frac{n(n^2+3)}{3}$ (B) $\frac{n(n^2+2)}{3}$
(C) $\frac{n(n^2+1)}{3}$ (D) None of these

Q.8 The region of Argand diagram defined by $|z - 1| + |z + 1| \le 4$ is-

(A) interior of an ellipse
(B) exterior of a circle
(C) interior and boundary of an ellipse
(D) None of these

Q.9 The roots of the cubic equation $(z + ab)^3 = a^3$, $a \ne 0$ represents the vertices of an equilateral triangle of sides of length-

(A) $\frac{1}{\sqrt{3}}|ab|$ (B) $\sqrt{3}|a|$
(C) $\sqrt{3}|b|$ (D) $\frac{1}{\sqrt{3}}|a|$

Q.10 Locus of the point z satisfying the equation |iz – 1| + |z – i| = 2 is-

(A) a straight line (B) a circle
(C) an ellipse (D) a pair of straight lines

Q.11 If 1, ω, ω^2 are the three cube roots of unity and α, β and γ are the cube roots of p, p < 0, then for any x, y and z the expression $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha} =$

(A) 1 (B) ω
(C) ω^2 (D) None of these

Assertion & Reason type question :-

Each of the questions given below consists of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement- I and Statement- II are true, and Statement-II is the correct explanation of Statement– I.

(B) If both Statement - I and Statement –II are true but Statement - II is not the correct explanation of Statement – I.

(C) If Statement-I is true but Statement-II is false.

(D) If Statement-I is false but Statement-II is true.

Q.12 **Statement I :** The expression $\left(\frac{2i}{1+i}\right)^n$ is a positive integer for all values of n.

Statement II : Here n = 8 is the least positive for which the above expression is a positive integer.

Q.13 **Statement I :** We have an equation involving the complex number z is $\left|\frac{z-3i}{z+3i}\right| = 1$ which lies on the x-axis.

Statement II :

The equation of the x-axis is y = 3

Q.14 **Statement I :**

If $|z| < \sqrt{2} - 1$, then $|z^2 + 2z\cos\alpha| < 1$.

Statement II :

$|z_1 + z_2| \le |z_1| + |z_2|$, also $|\cos\alpha| \le 1$.

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 Let z and w are two non zero complex number such that |z| = |w| , and Arg (z) + Arg (w) = π then - **[AIEEE-2002, IIT-1995]**

(A) z = w (B) $z = \overline{w}$

(C) $\overline{z} = \overline{w}$ (D) $z = -\overline{w}$

Q.2 If |z − 2| ≥ |z − 4| then correct statement is- **[AIEEE-2002]**

(A) R (z) ≥ 3 (B) R(z) ≤ 3

(C) R(z) ≥ 2 (D) R(z) ≤ 2

Q.3 If z and ω are two non- zero complex numbers such that |zω| = 1, and Arg(z) – Arg(ω) $= \frac{\pi}{2}$, then $\overline{z}\,\omega$ is equal to- **[AIEEE - 2003]**

(A) – i (B) 1

(C) – 1 (D) i

Q.4 Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further assume that the origin, z_1 and z_2 form an equilateral triangle. Then **[AIEEE-2003]**

(A) $a^2 = 4b$ (B) $a^2 = b$

(C) $a^2 = 2b$ (D) $a^2 = 3b$

Q.5 If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **[AIEEE - 2003]**

(A) x = 2n + 1, where n is any positive integer

(B) x = 4n, where n is any positive integer

(C) x = 2n, where n is any positive integer

(D) x = 4n + 1, where n is any positive integer

Q.6 Let z, w be complex numbers such that $\overline{z} + i\overline{w} = 0$ and arg zw = π. Then arg z equals- **[AIEEE - 2004]**

(A) π/4 (B) π/2

(C) 3 π/4 (D) 5 π/4

Q.7 If z = x – iy and $z^{1/3} = p + iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to- **[AIEEE - 2004]**

(A) 1 (B) – 1

(C) 2 (D) – 2

Q.8 If $|z^2 - 1| = |z|^2 + 1$, then z lies on- **[AIEEE - 2004]**

(A) the real axis (B) the imaginary axis

(C) a circle (D) an ellipse

Q.9 If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to- **[AIEEE - 2005]**

(A) $\frac{\pi}{2}$ (B) –π (C) 0 (D) $\frac{-\pi}{2}$

Q.10 If $w = \frac{z}{z - \frac{1}{3}i}$ and | w | = 1, then z lies on- **[AIEEE - 2005]**

(A) an ellipse (B) a circle

(C) a straight line (D) a parabola

Q.11 If |z + 4| ≤ 3, then the maximum and minimum value of |z + 1| are- **[AIEEE - 2007]**

(A) 4, 1 (B) 4, 0

(C) 6, 0 (D) 6, 1

Q.12 The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is- **[AIEEE - 2008]**

(A) $\frac{1}{i+1}$ (B) $\frac{-1}{i+1}$

(C) $\frac{1}{i-1}$ (D) $\frac{-1}{i-1}$

Q.13 If ω is an imaginary cube root of unity then $(1+\omega-\omega^2)(1+\omega^2-\omega)$ equals- **[AIEEE - 2002]**

(A) 0 (B) 1 (C) 2 (D) 4

Q.14 If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3+8=0$, are - **[AIEEE-2005]**

(A) $-1, -1+2\omega, -1-2\omega^2$
(B) $-1, -1, -1$
(C) $-1, 1-2\omega, 1-2\omega^2$
(D) $-1, 1+2\omega, 1+2\omega^2$

Q.15 If $z^2+z+1=0$, where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+......+\left(z^6+\frac{1}{z^6}\right)^2$ is – **[AIEEE 2006]**

(A) 54 (B) 6 (C) 12 (D) 18

Q.16 Let **A** and **B** denote the statements
A : $\cos\alpha+\cos\beta+\cos\gamma=0$
B : $\sin\alpha+\sin\beta+\sin\gamma=0$
If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$
then : **[AIEEE-2009]**
(A) A is false and B is true
(B) both A and B are true
(C) both A and B are false
(D) A is true and B is false

Q.17 If $\left|Z-\frac{4}{z}\right|=2$, then the maximum value of | Z | is equal to: **[AIEEE 2009]**

(A) $\sqrt{5}+1$ (B) 2
(C) $2+\sqrt{2}$ (D) $\sqrt{3}+1$

Q.18 The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals – **[AIEEE 2010]**

(A) 0 (B) 1 (C) 2 (D) ∞

Q.19 If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7=A+B\omega$. Then (A, B) equals – **[AIEEE 2011]**

(A) (0, 1) (B) (1, 1)
(C) (2, 0) (D) (–1, 1)

Q.20 Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re z = 1, then it is necessary that : **[AIEEE 2011]**

(A) $\beta\in(0,1)$ (B) $\beta\in(-1,0)$
(C) $|\beta|=1$ (D) $\beta\in(1,\infty)$

Q.21 If z ≠ 1 and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :
(A) on a circle with centre at the origin.
(B) either on the real axis or on a circle not passing through the origin.
(C) on the imaginary axis.
(D) either on the real axis or on a circle passing through the origin. **[AIEEE 2012]**

Q.22 If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals – **[JEE Main - 2013]**

(A) θ (B) π – θ (C) – θ (D) $\frac{\pi}{2}-\theta$

SECTION-B

Q.1 The equation not representing a circle is given by - **[IIT - 1991]**

(A) $R_e\left(\frac{1+z}{1-z}\right)=0$ (B) $z\bar{z}+iz-i\bar{z}+1=0$
(C) $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$ (D) $\left|\frac{z-1}{z+1}\right|=1$

Q.2 If z is a complex number such that z ≠ 0 and $R_e(z)=0$, then- **[IIT - 1992]**

(A) $R_e(z^2)=0$ (B) $I_m(z^2)=0$
(C) $R_e(z^2)=I_m(z^2)$ (D) none of these

Q.3 If α and β are different complex numbers with |β| = 1, then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is equal to - **[IIT - 1992]**

(A) 0 (B) 1/2
(C) 1 (D) 2

Q.4 The smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$ is - **[IIT - 1993]**

(A) 4 (B) 8
(C) 2 (D) 12

Q.5 If $z_1 = 8 + 4i$, $z_2 = 6 + 4i$ and $\arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$, then z satisfies- **[IIT - 1993]**

(A) $|z - 7 - 4i| = 1$ (B) $|z-7-5i| = \sqrt{2}$
(C) $|z - 4i| = 8$ (D) $|z-7i| = \sqrt{18}$

Q.6 if ω is an imaginary cube root of unity, then the value of $\sin\left[(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right]$ is- **[IIT - 1994]**

(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{\sqrt{3}}{2}$

Q.7 If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and If $z_1 = 1 + i\sqrt{3}$, then - **[IIT-1994, 1999]**

(A) $z_2 = -2$, $z_3 = 1 - i\sqrt{3}$
(B) $z_2 = 2$, $z_3 = 1 - i\sqrt{3}$
(C) $z_2 = -2$, $z_3 = -1 - i\sqrt{3}$
(D) $z_2 = -1 - i\sqrt{3}$, $z_3 = -1 - i\sqrt{3}$

Q.8 If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A & B are respectively the numbers - **[IIT - 1995]**

(A) 0, 1 (B) 1, 1
(C) 1, 0 (D) –1, 1

Q.9 $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then- **[IIT-1998]**

(A) x = 3 , y = 1 (B) x = 1 , y = 3
(C) x = 0 , y = 3 (D) x = 0 , y = 0

Q.10 If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals **[IIT - 1998]**

(A) 128 ω (B) –128 ω
(C) $128\,\omega^2$ (D) $-128\,\omega^2$

Q.11 The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals **[IIT- 1998]**

(A) i (B) i – 1
(C) – i (D) 0

Q.12 If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to- **[IIT-1999]**

(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$
(C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

Q.13 If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is - **[IIT - 2000]**

(A) equal to 1 (B) less than 1
(C) greater than 3 (D) equal to 3

Q.14 If arg (z) < 0, then arg (–z) – arg (z) = **[IIT - 2000]**

(A) π (B) –π (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

Q.15 The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangles which is - **[IIT - 2001]**

(A) of area zero
(B) right angled isosceles
(C) equilateral
(D) obtuse angled isosceles

Q.16 For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is - **[IIT - 2002]**

(A) 0 (B) 2 (C) 7 (D) 17

Q.17 If $|z| = 1$, $z \neq -1$ and $w = \frac{z-1}{z+1}$ then real part of w = ? **[IIT Scr- 2003]**

(A) $\frac{-1}{|z+1|^2}$ (B) $\frac{1}{|z+1|^2}$

(C) $\frac{2}{|z+1|^2}$ (D) 0

Q.18 If ω is cube root of unity (ω ≠ 1) then the least value of n, where n is positive integer such that $(1 + \omega^2)^n = (1 + \omega^4)^n$ is - **[IIT - Sc-2004]**

(A) 2 (B) 3 (C) 5 (D) 6

Q.19 A man walks a distance of 3 units from the origin towards the north-east (N 45º E) direction. From there, he walks a distance of 4 units towards the north-west (N 45º W) direction to reach a point P. Then the position of P in the Argand plane is- **[IIT - 2007]**

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)\ e^{i\pi/4}$

(C) $(4 + 3i)\ e^{i\pi/4}$ (D) $(3 + 4i)\ e^{i\pi/4}$

Q.20 If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on- **[IIT - 2007]**

(A) a line not passing through the origin

(B) $|z| = \sqrt{2}$

(C) the x-axis

(D) the y-axis

Q.21 Let z = x + iy be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is- **[IIT - 2009]**

(A) 48 (B) 32 (C) 40 (D) 80

Q.22 The set

$\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right);\ z \text{ is a complex number}, |z| = 1, z \neq \pm 1\right\}$ is -

[IIT - 2011]

(A) $(-\infty, -1) \cup (1, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$

(C) $[2, \infty)$ (D) $(-\infty, -1] \cup [1, \infty)$

Q.23 Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value

[IIT – 2012]

(A) – 1 (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Q.24 Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[JEE - Advance 2013]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

Q.25 Let $w = \frac{\sqrt{3}+1}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$.

Further $H_1 = \left\{z \in C : \text{Re}\, z > \frac{1}{2}\right\}$ and

$H_2 = \left\{z \in C : \text{Re}\, z < \frac{-1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

[JEE - Advance 2013]

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Paragraph for Questions 26 and 27

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z - 1 + \sqrt{3}\,i}{1 - \sqrt{3}\,i}\right] > 0\right\}$ and

$S_3 = \{z \in C : \text{Re}\, z > 0\}$

[JEE - Advance 2013]

Q.26 $\min\limits_{z \in S} |1 - 3i - z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

Q.27 Area of S =

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	D	B	D	D	C	B	A	C	C	B	A	B	B	D	B	B	B	B
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	D	D	B	B	D	A	A	B	A	D	A	D	A	A	B	B	A	A	B	D
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	D	B	D	C	B	A	D	D	D	B	A	C	B	C	B	C	B	B	B	B
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	C	B	B	B	D	A	B	B	B	C	B	B	A	A	C	C	B	D	A
Q.No.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	D	C	A	B	A	D	D	A	B	A	A	D	D	C	C	D	A	C	A	A
Q.No.	101	102																		
Ans.	B	C																		

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	C	D	B	C	D	B	C	C	D	B	D	B	A	C	D	B	B	C
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37			
Ans.	C	C	C	C	C	A	D	D	D	A	A	A	C	A	D	A	B			

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	D	B	C	A	D	B	C	B	A	C	D	C	A

LEVEL- 4

SECTION-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	A	A	D	B	C	D	B	C	C	C	B	D	C	C	B	A	B	B	D
Q.No.	21	22																		
Ans.	D	A																		

SECTION-B

1.[D] is straight line

2.[B] $z = iy$

$z^2 = -y^2$

$\therefore\ I_m(z^2) = 0$

3.[C] $|\beta| = 1 \Rightarrow \bar{\beta} = \dfrac{1}{\beta}$

$\Rightarrow \dfrac{|\beta-\alpha|}{|1-\bar{\alpha}\beta|} = \dfrac{|\beta-\alpha|}{\left|1-\dfrac{\bar{\alpha}}{\bar{\beta}}\right|}$

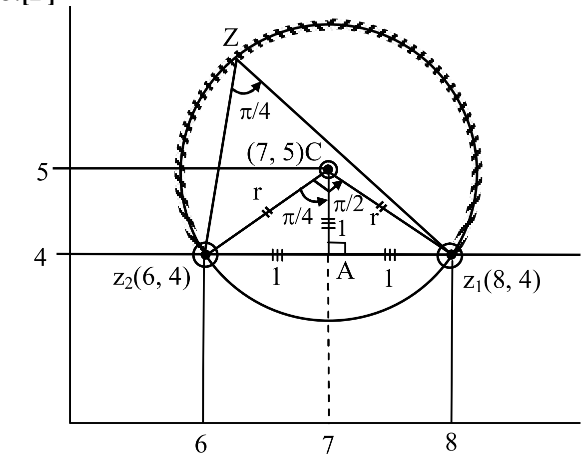
$= |\bar{\beta}|\,.\,\dfrac{|\beta-\alpha|}{|\bar{\beta}-\bar{\alpha}|} = |\bar{\beta}| = 1$

4.[C] $\left(\dfrac{1+i}{1-i}\right)^{2n} = 1$

$\Rightarrow i^{2n} = 1$

$\Rightarrow (-1)^n = 1$

5.[B]



$Az_2 = 1 \therefore AC = 1$

$\therefore r = \sqrt{2}$ & C(7, 5)

6.[C] $= \sin\left[(\omega + \omega^2)\pi - \frac{\pi}{4}\right] \qquad = \sin\left(-\pi - \frac{\pi}{4}\right)$

$= -\sin\left(\pi + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

7.[A] Centre C(0, 0) & r = 2

$z_2 = \omega z_1$ & $z_3 = \omega^2 z_1$

$\therefore z_2 = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(1 + i\sqrt{3})$

$z_2 = \frac{1}{2}(i\sqrt{3} - 1) \cdot (i\sqrt{3} + 1)$

$z_2 = \frac{1}{2}[(i\sqrt{3})^2 - (1)^2]$

$= \frac{1}{2}[-3 - 1] = -2$

& $z_3 = \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(1 + i\sqrt{3})$

$\Rightarrow z_3 = -\frac{1}{2}(1 + i\sqrt{3})^2 = -\frac{1}{2}(1 - 3 + 2i\sqrt{3})$

$\Rightarrow z_3 = -\frac{1}{2}(-2 + 2i\sqrt{3}) = 1 - i\sqrt{3}$

8.[B] $(-\omega^2)^7 = A + B\omega$

$\Rightarrow -\omega^2 = 1 + \omega = A + B\omega$

$\therefore A = B = 1$

9.[D] $\Rightarrow 6i(-3 + 3) + 3i(4i + 20) + 1(12 - 60i)$

$= 0 - 12 + 60i + 12 - 60i = 0$

$\therefore x = 0, y = 0$

10.[D] $(1 + \omega - \omega^2)^7 = (-2\omega^2)^7 = -128\omega^{14}$

11.[B] $\sum_{n=1}^{13}(i^n + i^{n+1}) = (1 + i)\sum_{n=1}^{13} i^n$

$= (1 + i)\, i^{13} = (1 + i)i = i - 1$

12.[C] $= 4 + 5\,\omega^{334} + 3\omega^{365}$

$= 4 + 5\omega + 3\omega^2$

$= 4 + 2\omega + \frac{(3\omega + 3\omega^2)}{-3}$

$= 1 + 2\omega$

$= 1 + 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = i\sqrt{3}$

13.[A] $|z| = 1 \Rightarrow \bar{z} = \frac{1}{2}$

$\because \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$

$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$

$\Rightarrow |z_1 + z_2 + z_3| = 1$

14.[A] Let Arg(z) = θ (θ < 0)

Then Arg(–z) = π + θ

$\because$ Arg(–z) – Arg(z) = π + θ – θ = π

15.[C] $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2} \Rightarrow \frac{2z_1 - 2z_3}{z_2 - z_3} - 1 = -i\sqrt{3}$

$\Rightarrow (2z_1 - z_2 - z_3) = -i\sqrt{3}\,(z_2 - z_3)$

squaring

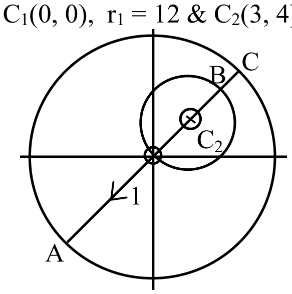
$\Rightarrow 4z_1^2 + z_2^2 + z_3^2 - 4z_1z_2 + 2z_2z_3 - 4z_1z_3$

$= -3(z_2^2 + z_3^2 - 2z_2z_3)$

$\Rightarrow 4z_1^2 + 4z_2^2 + 4z_3^2 - 4z_1z_2 - 4z_2z_3 - 4z_1z_3 = 0$

which is condition for equilateral Δ.

16.[B] $C_1(0, 0)$, $r_1 = 12$ & $C_2(3, 4)$, $r_2 = 5$



$|z_1 - z_2|_{min} = BC = C_1C - C_1B$

$= 12 - 10 = 2$

17.[D] $|z| = 1 \Rightarrow \bar{z} = \frac{1}{z}$

$\omega = \frac{z - 1}{z + 1}$

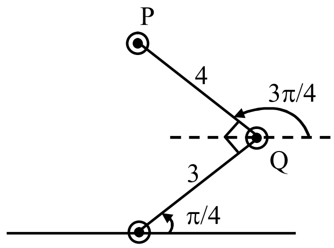
$\bar{\omega} = \frac{\bar{z} - 1}{\bar{z} + 1} = \frac{\frac{1}{z} - 1}{\frac{1}{z} + 1} \Rightarrow \bar{\omega} = \frac{1 - z}{1 + z}$

$\therefore \omega + \bar{\omega} = 0$

$\therefore Re(\omega) = 0$

18.[B] $(-\omega)^n = (-\omega^2)^n \Rightarrow \omega^n = 1$

19.[D]



$\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$

$= 3\text{cis}\frac{\pi}{4} + 4\text{ cis}\frac{3\pi}{4} = \text{cis}\frac{\pi}{4}[3 + 4\text{cis}\frac{\pi}{2}]$

$= e^{i\pi/4}(3 + 4i)$

20.[D] $|z| = 1 \Rightarrow \bar{z} = \frac{1}{z}$

$\Rightarrow \omega = \frac{z}{1-z^2}$

$\Rightarrow \bar{\omega} = \frac{\bar{z}}{1-\bar{z}^2} = \frac{1/z}{1-1/z^2} \Rightarrow \bar{\omega} = \frac{z}{z^2-1}$

$\therefore \omega + \bar{\omega} = 0$

w is purely imaginary

21.[A] $z\bar{z}^3 + \bar{z}z^3 = 350$

$\Rightarrow z\bar{z}(z^2 + \bar{z}^2) = 350$

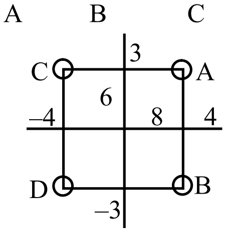
$\Rightarrow (x^2 + y^2)[2(x^2 - y^2)] = 350$

$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175$

$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 25\times9$

$\Rightarrow (x^2 + y^2) = 25$ & $x^2 - y^2 = 9$

Pts are (4, 3), (4, – 3), (– 4, 3), (– 4, –3)
A B C D



A = 48

22.[D] Let $z = \cos\theta + i\sin\theta$

so $\frac{2iz}{1-z^2} = \frac{2i(\cos+i\sin\theta)}{1-\cos2\theta - i\sin2\theta} = -\text{cos}ec\theta\ \forall\ \theta \neq (2n+1)\ \frac{\pi}{2}$

so $\text{Re}\left(\frac{2iz}{1-z^2}\right) = -\text{cosec}\,\theta \in (-\infty, -1] \cup [1,\ \infty)$

23.[D] put z = x + iy

$a = \left(z + \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because z \neq \frac{-1}{2}$ I(z) ≠ 0

$\therefore a \neq \frac{3}{4}$

24.[C] α lies on $|z - z_0| = r$

So $|\alpha - z_0| = r \Rightarrow |\alpha - z_0|^2 = r^2$ …(i)

$\frac{1}{\alpha}$ lies on $|z - z_0| = 2r$, So $\left|\frac{1}{\alpha} - z_0\right| = 2r$

$\Rightarrow |1 - \bar{\alpha}z_0| = 2r|\bar{\alpha}| \Rightarrow |1 - \bar{\alpha}z_0| = 2r|\alpha|$

$\Rightarrow |1 - \bar{\alpha}z_0|^2 = 4r^2|\alpha|^2$ …(ii)

Subtract (ii) from (i)

$|1 - \bar{\alpha}z_0|^2 - |\alpha - z_0|^2 = r^2(4|\alpha|^2 - 1)$

$\Rightarrow 1 + |\alpha|^2|z_0|^2 - |\alpha|^2 - |z_0|^2 = r^2(4|\alpha|^2 - 1)$

$\Rightarrow (1 - |\alpha|^2)(1 - |z_0|^2) - r^2(4|\alpha|^2 - 1) = 0$

$\Rightarrow (1 - |\alpha|^2)(1 - |z_0|^2) + 2(1 - |z_0|^2)(4|\alpha|^2 - 1) = 0$

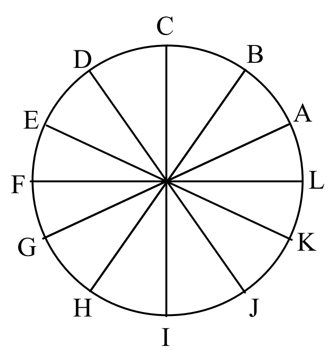
$\Rightarrow (1 - |z_0|^2)(1 - |\alpha|^2 + 8|\alpha|^2 - 2) = 0$

$\Rightarrow (1 - |z_0|^2)(7|\alpha|^2 - 1) = 0$

$\Rightarrow |\alpha|^2 = 1/7 \quad \Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$

25. [C,D] $\omega = \frac{\sqrt{3}+i}{2}$

Powers of ω lies on a unit circle centred at origin lying at a difference of angle $\frac{\pi}{6}$



Now for H_1 $\text{Re}(z) > \frac{1}{2}$

So $P \cap H_1$ can be at point A, L, K

For H_2 $\text{Re}(z) < -\frac{1}{2}$

So $P \cap H_2$ can be at point E, F, G

So $\angle z_1Oz_2$ can be $\frac{2\pi}{3}, \frac{5\pi}{6}$

26.[C] $S_1 : x^2 + y^2 \le 16$

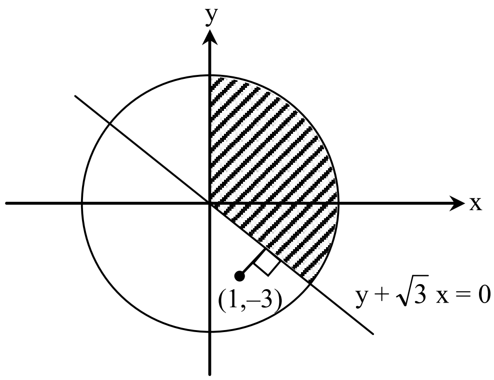
$S_2 : \text{Img}\left(\dfrac{(x-1)+i(y+\sqrt{3})}{1-i\sqrt{3}}\right) > 0$

$\Rightarrow \sqrt{3}\,(x-1) + y + \sqrt{3} > 0$

$\Rightarrow \sqrt{3}\,x + y > 0$

$S_3 : x > 0$

Shaded area represents 'S'



Now min | 1 –3i – z | = min | z – 1 + 3i |

= minimum distance from (1, – 3)

Perpendicular distance of (1, – 3) from line $y + \sqrt{3}\,x = 0$

$= \dfrac{3-\sqrt{3}}{2}$

27.[B] Area of S $= \dfrac{1}{2} \times (4)^2 \times \dfrac{5\pi}{6}$

$= \dfrac{20\pi}{3}$